

Dragged metrics

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We show that the path of any accelerated body in an arbitrary space-time geometry $g_{\mu\nu}$ can be described as geodesics in a dragged metric $\hat{q}_{\mu\nu}$ that depends only on the background metric and on the motion of the body. Such procedure allows the interpretation of all kind of non-gravitational forces as modifications of the metric of space-time. This method of effective elimination of the forces by a change of the metric of the substratum can be understood as a generalization of the d'Alembert principle applied to all relativistic processes.

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I. INTRODUCTION

In 1923 Gordon [1] made a seminal suggestion to treat the propagation of electromagnetic waves in a moving dielectric as a modification of the metric structure of the background. He showed that the waves propagate as geodesics not in the geometry $\eta_{\mu\nu}$ but instead in the dragged metric

$$\hat{q}^{\mu\nu} = \eta^{\mu\nu} + (\epsilon\mu - 1) v^\mu v^\nu, \quad (1)$$

where ϵ and μ are constant parameters that characterize the dielectric. Latter it was recognized that this interpretation can be used to describe non-linear structures when ϵ and μ depends on the intensity of the field [2].

In recent years an intense activity concerning properties of Riemannian geometries similar to the one described by Gordon has been done [3]. In particular those that allows a binomial form for both the metric and its inverse, that is its covariant and the corresponding contravariant expressions

$$\hat{q}^{\mu\nu} = \eta^{\mu\nu} + b \Phi^{\mu\nu}, \quad (2)$$

and

$$\hat{q}_{\mu\nu} = A \eta_{\mu\nu} + B \Phi_{\mu\nu}. \quad (3)$$

Thus, the tensor $\Phi^{\mu\nu}$ must satisfy the condition

$$\Phi_{\mu\nu} \Phi^{\nu\lambda} = m \delta_\mu^\lambda + n \Phi_\mu^\lambda. \quad (4)$$

In the present paper we limit our analysis only to the simplest dragged form by setting $\Phi^{\mu\nu} = v^\mu v^\nu$. In this case the coefficients of the covariant form of the metric are given by

$$A = 1; \quad B = -\frac{b}{1+b}.$$

The origin of the dragging effect in the case of Gordon's metric is due to the modifications of the path of the electromagnetic waves inside the moving dielectric. Then we face the question: could such particular description of the electromagnetic waves in moving bodies be generalized for other cases, independently of the electromagnetic forces? In other words, could such geometrized paths be used to describe other kinds of forces? We shall see that the answer is yes. Indeed, we will show that it is possible to geometrize different kinds of forces by the introduction of a dragged metric $\hat{q}_{\mu\nu}$ such that in this geometry the accelerated body follows the free path of geodesics.

Let us emphasize that we deal here with any kind of force that has a non-gravitational character. It is precisely the consequences of such non-gravitational force that we describe in terms of a modified dragged metric. This means that the observable effects of any force can be interpreted as nothing but a modification of the geometry of space-time. In other words the motion of any accelerated body can be described as a free body following geodesics in a modified metric. This procedures generalizes d'Alembert principle of classical mechanics [4, 5] which states that it is possible to transform a dynamical problem into a static one, where the body is free of any force. Going from the background metric – where an accelerated body experiences a non-gravitational force – to a dragged metric where the body follows a geodesics and become free of non-gravitational forces is the relativistic expression of this principle. In this way we produce a geometric description of all kind of motion whatever the force that originates it.

II. A SPECIAL CASE

We claim that accelerated bodies in a flat Minkowski space-time¹ can be equivalently described as free bodies

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¹ Let us point out that the analysis we present in this section may be straightforwardly generalized to arbitrary curved Riemannian background.

following geodesics in an associated dragged geometry. In order to simplify our calculation we restrict this section to the case in which the acceleration vector a_μ is the gradient of a function, that is²

$$a_\mu = \partial_\mu \Psi. \quad (5)$$

Thus the force acting on the body under observation comes from a potential V in the Lagrangian formalism, i.e., $F_\mu = \partial_\mu V$.

We write the dragged metric in the form

$$\hat{q}^{\mu\nu} = \eta^{\mu\nu} + b v^\mu v^\nu.$$

The associated covariant derivative is defined by

$$v^\alpha{}_{;\mu} = v^\alpha{}_{,\mu} + \hat{\Gamma}^\alpha_{\mu\nu} v^\nu,$$

where the corresponding Christoffel symbol is given by

$$\hat{\Gamma}^\epsilon_{\mu\nu} = \frac{1}{2} (\eta^{\epsilon\alpha} + b v^\epsilon v^\alpha) (\hat{q}_{\alpha\mu,\nu} + \hat{q}_{\alpha\nu,\mu} - \hat{q}_{\mu\nu,\alpha}), \quad (6)$$

where we are using a comma to denote simple derivative, that is $A_{,\mu} \equiv \partial_\mu A$. The description of an accelerated curve in a flat space-time as a geodesics in a dragged metric is possible if the following condition is satisfied

$$(v_{\mu,\nu} - \hat{\Gamma}^\epsilon_{\mu\nu} v_\epsilon) \hat{v}^\nu = 0, \quad (7)$$

where we have used the dragged metric to write $\hat{v}^\mu \equiv \hat{q}^{\mu\nu} v_\nu$. Or, equivalently,

$$(v_{\mu,\nu} - \hat{\Gamma}^\epsilon_{\mu\nu} v_\epsilon) v^\nu = 0. \quad (8)$$

Then noting that the acceleration in the background is defined by $a_\mu = v_{\mu,\nu} v^\nu$ and using equation (5) the condition of geodesics in the dragged geometry takes the form

$$\partial_\mu \Psi = \hat{\Gamma}^\epsilon_{\mu\nu} v_\epsilon v^\nu. \quad (9)$$

We have

$$\hat{\Gamma}^\epsilon_{\mu\nu} v_\epsilon v^\nu = \frac{1+b}{2} v^\alpha v^\nu \hat{q}_{\alpha\nu,\mu}. \quad (10)$$

Using the expression of $\hat{q}_{\alpha\nu}$ and combining with condition (8) it follows

$$a_\mu + \frac{\partial_\mu b}{2(1+b)} = 0,$$

that is, the expression of the coefficient b of the dragged metric is given in terms of the potential of the acceleration

$$1+b = e^{-2\Psi}. \quad (11)$$

A. The curvature of the dragged metric

In the case the background metric is not flat or if we use an arbitrary coordinate system the connection is given by the sum of the corresponding background one and a tensor, that is

$$\hat{\Gamma}^\epsilon_{\mu\nu} = \Gamma^\epsilon_{\mu\nu} + K^\epsilon_{\mu\nu}. \quad (12)$$

In the case of the Minkowski background a direct calculation gives for the connection $\hat{\Gamma}^\epsilon_{\mu\nu}$ the form

$$K^\epsilon_{\mu\nu} = v^\epsilon (a_\mu v_\nu + a_\nu v_\mu) - a^\epsilon v_\mu v_\nu. \quad (13)$$

Then,

$$K^\epsilon_{\mu\epsilon} = a_\mu.$$

The contracted Ricci curvature has the expression

$$\hat{R}_{\mu\nu} = a_{\mu,\nu} - a_\mu a_\nu + (\omega + a^\alpha{}_{,\alpha}) v_\mu v_\nu, \quad (14)$$

Noting that $\hat{a}^\mu = a_\nu \hat{q}^{\mu\nu} = a^\mu$ it follows that

$$\omega \equiv a^\mu a_\mu = a_\mu a_\nu \eta^{\mu\nu} = a_\mu a_\nu \hat{q}^{\mu\nu} = \hat{\omega}.$$

The scalar of curvature $\hat{R} = \hat{R}_{\mu\nu} \hat{q}^{\mu\nu}$ is

$$\hat{R} = (2+b) a^\alpha{}_{,\alpha}. \quad (15)$$

These expressions can be re-written in a covariant way by noting that

$$a_{\mu;\nu} \equiv a_{\mu,\nu} - \hat{\Gamma}^\epsilon_{\mu\nu} a_\epsilon,$$

which yields

$$\hat{R}_{\mu\nu} = a_{\mu;\nu} - a_\mu a_\nu - (\omega - a^\alpha{}_{;\alpha}) v_\mu v_\nu, \quad (16)$$

and for the scalar \hat{R} the form

$$\hat{R} = (2+b) (a^\alpha{}_{;\alpha} - \omega). \quad (17)$$

B. Analog gravity

Suppose that an observer following a path with four-velocity v_μ and acceleration a_μ in the flat Minkowski space-time background is not able to identify the origin of the force that is acting on him. In other words he is going to believe that only long-range gravitational forces are constraining his motion. Let us assume that he knows that gravity does not accelerate any curve but instead change the metric of the background according to the principles of general relativity. This means that if he is able to represent his motion as a geodesics in a dragged

² Using the freedom in the definition of the four-vector v_μ we set $v_\mu v^\mu = 1$. The acceleration is orthogonal to it, that is, $a_\mu v^\mu = 0$.

metric $\hat{q}_{\mu\nu}$ he will consider that the origin of such curved metric is nothing but a consequence of a distribution of energy which he will describe by using the equation

$$\hat{R}_{\mu\nu} - \frac{1}{2} \hat{R} \hat{q}_{\mu\nu} = -\hat{T}_{\mu\nu}. \quad (18)$$

He will identify the different terms of the source through his own motion. From his velocity v_μ he defines the normalized four-velocity \hat{u}^α in the \hat{Q} metric by setting

$$\hat{u}^\alpha = \sqrt{1 + b} v^\alpha.$$

He then proceed to characterize the origin of the curved metric using the standard decomposition

(a) density of energy

$$\hat{\rho} = \hat{T}_{\mu\nu} \hat{u}^\mu \hat{u}^\nu; \quad (19)$$

(b) isotropic pressure

$$\hat{p} = -\frac{1}{3} \hat{T}_{\mu\nu} h^{\mu\nu}; \quad (20)$$

(c) heat flux

$$\hat{q}_\lambda = \hat{T}_{\alpha\beta} \hat{u}^\beta h^\alpha_\lambda; \quad (21)$$

(d) anisotropic pressure

$$\hat{\pi}_{\mu\nu} = \hat{T}_{\alpha\beta} h^\alpha_\mu h^\beta_\nu + \hat{p} h_{\mu\nu}. \quad (22)$$

In these expressions we used

$$\hat{h}_{\mu\nu} = \hat{q}_{\mu\nu} - \hat{u}_\mu \hat{u}_\nu.$$

Note that $\hat{h}_{\mu\nu} = h_{\mu\nu}$. Thus, he will write

$$\hat{T}_{\mu\nu} = \hat{\rho} \hat{u}_\mu \hat{u}_\nu - \hat{p} h_{\mu\nu} + \hat{q}_\mu \hat{u}_\nu + \hat{q}_\nu \hat{u}_\mu + \hat{\pi}_{\mu\nu}. \quad (23)$$

From this decomposition, using equation (18) and the curvature (16) he will identify the energy-momentum distribution as

$$\begin{aligned} \hat{\rho} &= \frac{b}{2} Q, \\ \hat{p} &= \left(\frac{2}{3} + \frac{b}{2}\right) Q, \\ \hat{q}_\mu &= 0, \\ \hat{\pi}_{\mu\nu} &= -a_{\mu;\nu} + a_\mu a_\nu - \frac{Q}{3} \hat{q}_{\mu\nu} + \frac{Q}{3} \hat{u}_\mu \hat{u}_\nu, \end{aligned} \quad (24)$$

where

$$Q = \omega - a^\alpha_{;\alpha},$$

Summarizing, we can say that this observer will state that there is a gravitational field represented by the metric $\hat{q}_{\mu\nu}$ produced by the distribution of energy given

by equation (24). We note that this reduction of the dragged metric to the framework of general relativity is not mandatory. Indeed, we deal here precisely with some accelerated paths that are not reduced to the gravitational force in the standard theory. This will become more clear when we present examples of accelerated curves in specific solutions of general relativity in the next sections.

Indeed, let us present some clarifying examples. The first one considers the motion of rotating bodies in Minkowski space-time. The other ones take into account the general relativity effects. We shall see that it is possible to produce what could be called *double gravity*, if the origin of the curvature of the dragged metric is identified to an effective energy-momentum tensor satisfying the equations of general relativity. However there is no reason for this restriction. We will come back to this question elsewhere.

III. ACCELERATION IN MINKOWSKI SPACE-TIME

Let us consider a simple example concerning the acceleration of a body in flat Minkowski space-time written in non-stationary cylindrical coordinate system (t, r, ϕ, z) to express the following line element

$$ds^2 = a^2[dt^2 - dr^2 - dz^2 + g(r)d\phi^2 + 2h(r)d\phi dt], \quad (25)$$

where a is a constant. We choose the following local tetrad frame given implicitly by the 1-forms

$$\begin{aligned} \theta^0 &= a(dt + h d\phi), \\ \theta^1 &= a dr, \\ \theta^2 &= a \Delta d\phi, \\ \theta^3 &= a dz, \end{aligned} \quad (26)$$

where we define $\Delta = \sqrt{h^2 - g}$. The unique non-identically null components of the Ricci tensor R_{AB} in the tetrad frame are

$$\begin{aligned} R_{00} &= \frac{1}{2a^2} \left(\frac{h'}{\Delta} \right), \\ R_{11} &= \frac{1}{a^2} \left(\frac{\Delta''}{\Delta} - \frac{1}{2} \frac{h'^2}{\Delta^2} \right) = R_{22} = R_{33}, \\ R_{02} &= \frac{1}{2a^2} \left(-\frac{h''}{\Delta} + \frac{h'\Delta}{\Delta^2} \right). \end{aligned} \quad (27)$$

where a prime means derivative with respect to coordinate r . The equations of general relativity for this geometry have two simple solutions that we shall analyze below.

In the case of $R_{AB}=0$, we get

$$h' = 0; \quad \Delta'' = 0.$$

Solving these equations, we find

$$h \equiv \text{const}; \quad \Delta \equiv \omega^2 r^2,$$

where ω is a constant. Therefore, Eq. (25) takes the form

$$ds^2 = a^2[dt^2 - dr^2 - dz^2 + (h^2 - \omega^2 r^2)d\phi^2 + 2h(r)d\phi dt]. \quad (28)$$

If we consider the observer field

$$v^\mu = \frac{1}{a\sqrt{h^2 - \omega^2 r^2}}\delta_2^\mu.$$

This path corresponds to an acceleration given by

$$a_\mu = \left(0, \frac{\omega^2 r}{(h^2 - \omega^2 r^2)}, 0, 0\right).$$

This means that $a_\mu = \partial_\mu \Psi$, where

$$2\Psi = -\ln(h^2 - \omega^2 r^2).$$

We are in a situation similar to the previous section since the acceleration is a gradient. The parameter b of the dragged metric is given by the expression (11)

$$1 + b = h^2 - \omega^2 r^2,$$

and for the dragged metric the form

$$\begin{aligned} \frac{ds^2}{a^2} &= \frac{\omega^4 r^4 - \omega^2 r^2 h^2 + 1}{(h^2 - \omega^2 r^2)^2} dt^2 + d\phi^2 \\ &+ \frac{2h}{h^2 - \omega^2 r^2} d\phi dt - dr^2 - dz^2 \end{aligned} \quad (29)$$

Note that we are dealing with the case in which $h^2 - \omega^2 r^2 > 0$. This allows the presence of accelerated closed time-like curves (CTC) in the original background that will be mapped into closed time-like geodesics (CTG) in the dragged geometry. Note that there exists a real singularity in $r = h/\omega$ and that \hat{q}_{00} change sign where

$$\omega^2 r_\pm^2 = \frac{h^2 \pm \sqrt{h^4 - 4}}{2}.$$

IV. ACCELERATION IN CURVED SPACE-TIMES

Let us present now how our dragged metric approach works in some solutions of the equations of general relativity. We choose three well-known geometries: Schwarzschild, Gödel universe and the Kerr solution. In these Riemannian manifolds we analyze some examples of accelerated paths that are interpreted as geodesics in the associated dragged metrics.

A. Schwarzschild geometry

We set the Schwarzschild metric in the (t, r, θ, φ) coordinate system

$$ds^2 = \left(1 - \frac{r_H}{r}\right) dt^2 - \frac{1}{\left(1 - \frac{r_H}{r}\right)} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (30)$$

Choose the path described by the four-velocity

$$v^\mu = \sqrt{1 - \frac{r_H}{r}} \delta_0^\mu.$$

The corresponding acceleration is

$$a_\mu = \left(0, -\frac{r_H}{2(r^2 - r r_H)}, 0, 0\right).$$

In this case the acceleration is the gradient of function Ψ given by

$$\Psi = -\frac{1}{2} \ln\left(1 - \frac{r_H}{r}\right).$$

The factor b of the dragged metric is given by

$$b = -\frac{r_H}{r},$$

and the dragged metric takes the form

$$ds^2 = dt^2 - \frac{1}{\left(1 - \frac{r_H}{r}\right)} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (31)$$

The only non-null Ricci curvature of this $\hat{q}^{\mu\nu}$ metric are

$$R_1^1 = \frac{r_H}{r^3};$$

$$R_2^2 = R_3^3 = -\frac{1}{2} R_1^1.$$

All 14 Debever invariants are finite in all points except at the origin $r = 0$.

B. Gödel's geometry

Let us now turn our analysis to the Gödel geometry. In the cylindrical coordinate system this metric is given by Eq. (25), where a is a constant related to the vorticity $a = 2/\omega^2$ and

$$h(r) = \sqrt{2} \sinh^2 r;$$

$$g(r) = \sinh^2 r (\sinh^2 r - 1).$$

For completeness we note the non-trivial contravariant terms of the metric:

$$\begin{aligned} g^{00} &= \frac{1 - \sinh^2 r}{a^2 \cosh^2 r}, \\ g^{02} &= \frac{\sqrt{2}}{a^2 \cosh^2 r}, \\ g^{22} &= \frac{-1}{a^2 \sinh^2 r \cosh^2 r}. \end{aligned} \quad (32)$$

In [6] it was pointed out the acausal properties of a particle moving into a circular orbit around the z -axis with four-velocity

$$v^\mu = \left(0, 0, \frac{1}{a \sinh r \sqrt{\sinh^2 r - 1}}, 0 \right).$$

This path corresponds to an acceleration given by

$$a^\mu = \left(0, \frac{\cosh r [2 \sinh^2 r - 1]}{a^2 \sinh r [\sinh^2 r - 1]}, 0, 0 \right).$$

This means that $a_\mu = \partial_\mu \Psi$, where

$$\Psi = -\ln(\sinh r \sqrt{\sinh^2 r - 1}).$$

Again, we are in a situation where the acceleration is a gradient. Therefore, the parameter b of the dragged metric is given by the expression (11) which in the Gödel's background takes the form

$$1 + b = \sinh^2 r (\sinh^2 r - 1),$$

and the dragged metric has the following form

$$\begin{aligned} \frac{d\hat{s}^2}{a^2} &= \frac{3 - \sinh^4 r}{(\sinh^2 r - 1)^2} dt^2 + d\phi^2 + 2 \frac{\sqrt{2}}{\sinh^2 r - 1} d\phi dt, \\ &- dr^2 - dz^2 \end{aligned} \quad (33)$$

From the analysis of geodesics in Gödel geometry the domain $r < r_c$ where $\sinh^2 r_c = 1$ separates causal from non-causal regions of the space-time. This is related to the fact that a geodesic that pass the value $r = 0$ will be confined within the domain Ω_i defined by the values of coordinate r in the region $0 < r < r_c$. See [7] for details. However, the gravitational field is finite in the region $r = r_c$. Nothing similar in the dragged metric, once at $\sinh^2 r = 1$ there exists a real singularity in the dragged metric. Only the exterior domain is allowed. This means that for this kind of accelerated path in Gödel geometry the allowed domain for the dragged metric is precisely the whole acausal region.

C. Kerr metric

Let us turn now to the dragged metric approach in the case the background is the Kerr metric. In the Boyer-Lindquist coordinate system this metric is given by

$$\begin{aligned} ds^2 &= \left(1 - \frac{2Mr}{\rho^2} \right) dt^2 - \frac{\rho^2}{\Sigma} dr^2 - \rho^2 d\theta^2 + \\ &+ \frac{4Mra \sin^2 \theta}{\rho^2} dt d\phi + \\ &- \left[(r^2 + a^2) \sin^2 \theta + \frac{2Mra^2 \sin^4 \theta}{\rho^2} \right] d\phi^2, \end{aligned} \quad (34)$$

where $\Sigma = r^2 + a^2 - 2Mr$ and $\rho^2 = r^2 + a^2 \cos^2 \theta$. On equatorial plane ($\theta = \pi/2$) consider the following vector field

$$v^\mu = \left(0, 0, 0, \frac{r}{\sqrt{-(r^2 + a^2)^2 + a^2 \Sigma}} \right).$$

This path corresponds to an acceleration given by

$$a_\mu = \left(0, -\frac{r^3 - Ma^2}{r^4 + r^2 a^2 + 2Mra^2}, 0, 0 \right).$$

This means that $a_\mu = \partial_\mu \Psi$, where

$$2\Psi = -\ln \left[- \left(r^2 + a^2 + \frac{2Ma^2}{r} \right) \right].$$

Once more we choose an accelerated path that can be represented by a gradient. The parameter b of the dragged metric is given by the expression (11)

$$1 + b = - \left(r^2 + a^2 + \frac{2Ma^2}{r} \right),$$

and for the dragged metric, on the equatorial plane, the form

$$\begin{aligned} \frac{d\hat{s}^2}{a^2} &= \frac{1}{(1+b)^2} \left(1 - \frac{2M}{r} - b \frac{4M^2 a^2}{r^2} \right) dt^2 + d\phi^2 \\ &+ \frac{4Ma}{(1+b)r} dt d\phi - \frac{r^2}{\Delta} dr^2, \end{aligned} \quad (35)$$

These two last cases (Gödel and Kerr) show a very curious and intriguing property: the accelerated CTC's at their respective metrics) are transformed in curves that are geodesics, that is CTG's (at their corresponding dragged metric). Besides, in both cases, the dragged metrics display a real singularity excluding the causal domain.

V. GENERAL CASE

In the precedent sections we limited our analysis to the case in which the acceleration is given by a unique function. Let us now pass to more general situation. In

order to geometrize any kind of force we must deal with a larger class of geometries. The most general form of dragged metric that allows the description of accelerated bodies as true geodesics in a modified geometry has the form

$$\hat{q}^{\mu\nu} = g^{\mu\nu} + b v^\mu v^\nu + m a^\mu a^\nu + n(v^\mu a^\nu + a^\mu v^\nu). \quad (36)$$

The three parameters b, m, n are related to the three degrees of freedom of the acceleration vector. The corresponding covariant form of the metric is given by

$$\hat{q}_{\mu\nu} = g_{\mu\nu} + B v_\mu v_\nu + M a_\mu a_\nu + N(v_\mu a_\nu + a_\mu v_\nu). \quad (37)$$

in which B, M, N are given in terms of the parameters b, m, n by the relations

$$B = -\frac{b(1+m\omega) - n^2\omega}{(1+b)(1+m\omega) - n^2\omega},$$

$$M = \frac{1}{1+m\omega} \left(-m + \frac{n^2}{(1+b)(1+m\omega) - n^2\omega} \right),$$

$$N = -\frac{n}{(1+b)(1+m\omega) - n^2\omega}.$$

In this case the equation that generalizes the condition of geodesics (8) has the form

$$a_\mu = \frac{1}{2} \left((1+b) v^\lambda v^\nu + n a^\lambda a^\nu \right) [\hat{q}_{\lambda\mu, \nu} + \hat{q}_{\lambda\nu, \mu} - \hat{q}_{\mu\nu, \lambda}]. \quad (38)$$

This equation can be cast in the following formal expression

$$a_\mu = \Psi_1 \partial_\mu b + \Psi_2 \partial_\mu m + \Psi_3 \partial_\mu n, \quad (39)$$

where each term Ψ_1, Ψ_2 and Ψ_3 depends on all three functions m, b and n .

Solving this equation for these three functions provide the most general expression for any acceleration.

With these results, we have transformed the path of any particle submitted to any kind of force as a geodetic motion in the dragged metric. This result is the extension of the d'Alembert principle, corresponding to all types of motion i.e. the acceleration is geometrized through the dragged metric.

VI. CONCLUSION

We summarize the novelty of our analysis in the following steps:

- Let v_μ represent the four-vector that describes the kinematics of a body in an arbitrary space-time endowed with a geometry $g_{\mu\nu}$;
- If the body experiences a non-gravitational force it acquires an acceleration a_μ ;
- It is always possible to define an associated dragged metric $\hat{q}_{\mu\nu}$ given by (37) such that in this metric the acceleration is removed. That is, the path is represented as a free particle that follows geodesics in the dragged geometry.

We have shown by a constructive operation that the following conjecture is true:

Conjecture. *For any accelerated path Γ described by four-vector velocity v_μ and acceleration a_μ in a given Riemannian geometry $g_{\mu\nu}$ we can always construct another geometry \hat{Q} endowed with a dragged metric $\hat{q}_{\mu\nu}$ which depends only on $g_{\mu\nu}, v_\mu$ and a_μ such that the path Γ is a geodesic in \hat{Q} .*

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